**Initial Report**

***Project progress so far***

As scheduled, I have finished reading some papers related to **sparse representation and recovery based** DOA approaches. The basics are comprehensively presented in [1]. This report presents the methodology of [1].

Consider the *K* narrowband signals arriving at an array of *M* sensors, formulated as

 (1)

where  denotes the received signals by an array of *M* sensors.  denotes the signal vector comprising of signal components from *K* independent or correlated sources, and is the array manifold matrix, the columns of which, , are called steering vectors. The parameter  represents the directions corresponding to the signal vector.

To cast the DOA problem as a sparse representation problem, an overcomplete matrix **A** is introduced in terms of all possible source directions , i.e. sampling grid. Then . The signal field is represented by a  vector in which  if the direction of the *k*-th source equals . Thus, the sparse representation of the DOA problem will be

 (2)

As a result, the ill-posed problem can be handled by solving the following optimization problem:

 (3)

In which the L-2 norm penalizes the noise residual and the regularization parameter  imposes a sparsity requirement on . It can be seen that by tuning the value of  a tradeoff can be achieved: when , no sparsity will be required, and when ,  will have all zero components. The rationale behind this sparse approach is that, compared with the possible directions, i.e., the size of the sampling grid, the number of sources, *K,* is small, so  will be sparse. To take all the snapshots of measurement at different times into consideration, (2) can be formulated as

 (4)

where , ,.

Since sparsity is only required in space domain instead of time, by defining  and penalizing the L-1 norm of , defined as, the optimization problem becomes

 (5)

in which denotes Frobenius norm. However, one significant drawback is the computational complexity of solving (5), which increases super-linearly with *T*. When *T* is large, this approach is not applicable in real-time DOA estimation. To handle this complexity problem, and L-1 SVD approach is proposed.

Taking SVD, we have . Since the signal subspace has at most a dimension of *K*, keep a reduced matrix , where . By setting  and , we obtain

 (6)

Now decompose (6) into columns, we get

 (7)

(7) has a similar structure as (3), but the number of equations are reduced significantly since *K* is much smaller than *T* in practice. Similarly, by defining

 (8)

 (9)

 (10)

the optimization problem becomes

 (11)

To utilize the standard SOC programming optimization techniques, re-formulate (11) as

 (12)

By solving this SOC problem, some sparse signal vectors are obtained, and the support of a vector corresponds to the source directions in the sampling grid.

Next step of the project will be to do MATLAB simulation of the sparse representation based DOA approach.

***References***

[1] Malioutov D, Cetin M, Willsky A S. **A sparse signal reconstruction perspective for source localization with sensor arrays**[J]. IEEE transactions on signal processing, 2005, 53(8): 3010-3022.